PREFACE

Daniel C. Mattis

It was the early 1960s. Elliott H. Lieb and the present author were colleagues at the IBM Research Laboratories in Yorktown Heights, occupying neighboring cubicles. We collaborated in the study, creation and solution of mathematical models of many-body phenomena. In the course of this work we ultimately came to the realization that solvable models – whether they dealt with interacting bosons, Heisenberg spins, "*X-Y*" spins, or whatever – typically shared an intrinsic property: all were restricted to one spatial dimension with no obvious generalizations to higher dimensions.

It seemed natural to exploit this commonality, although an initial idea of founding a "Journal of One-Dimensional Physics" was quickly dismissed once we became convinced there was insufficient material for more than a few issues. But a *book* entitled "Mathematical Physics in One Dimension"? [1] *That* should nail down the topic of "one-dimensional models," allow it to be indexed by name for the first time, and provide a convenient platform for future research. That is what we decided upon, and that's where the "Luttinger Model" came in. The topics, and the chapters, were boiled down to seven.

1) Classical statistical mechanics,
2) Disordered chains of harmonic oscillators,
3) Electron energy bands in ordered and disordered crystals,
4) The many-Fermion gas,
5) The Bose gas,
6) Magnetism,
7) Time-dependent phenomena.

[1] E.H. Lieb and D.C. Mattis, *Mathematical Physics in One Dimension*, Academic Press, New York, 1966

Each chapter opened with a presentation of difficulties that had been encountered in the named topic when confronting what is ordinarily an insoluble N-body problem ($N \rightarrow \infty$.) This was followed by a description of the specific methods (Bethe's ansatz, the transfer matrix, fermionization in the X-Y model, etc.) that had been devised to overcome these issues, and ended with reprints of what we considered to be the seminal papers solving the problem. We stumbled, however, when it came to chapter 4 dealing with the notoriously fractious many-Fermion problem. At the time the best-known contribution to this topic came from an approximately solved model[2] by Sin-itiro Tomonaga dating back to 1950. Then the present author recalled having seen Joaquin "Quin" Luttinger's contribution in the Journal of Mathematical Physics.[3] It bore the title "Exact Solution …" that seemingly fit the requirements. However, upon further examination, this "exact" solution did seem flawed in several respects – notably in its failure to satisfy the so-called "concavity theorem," its lack of a ground state, etc.

Clearly, Luttinger, unaware at the time either of Tomonaga's work or of Jordan's discoveries of a generation earlier,[4] had been attempting to transfer the Thirring model[5] to condensed matter physics without extensive re-examination. It was purely in the interest of completing this task for chapter 4 that we set about to understand his model – if that were possible – before finalizing the book. In the process we (re)discovered[4] "bosonization."[6]

Once we had figured it out, it was a bonus that our solution remained valid for arbitrary two-body interactions – not just delta function interactions – while still subject to the

[2] S. Tomonaga, Progr. Theoret. Phys. (Kyoto) **5**, 544 (1950)
[3] J.M. Luttinger, Math. Phys. **4**, 1154 (1963)
[4] In the prehistory of high-energy physics, P. Jordan had already proposed a theory of *light* (photons are bosons) starting from a one-dimensional field of *neutrinos* (fermions.) See P. Jordan, Z. Phys. **93**, 464 (1935), **98**, 759 (1936), **99**, 109 (1936), **102**, 243 (1936), **105**, 114 (1937), **105**, 229 (1937), *inter alia*.
[5] W. Thirring, Ann. Phys. (NY) **3**, 91 (1958)
[6] D.C. Mattis and E.H. Lieb, "*Exact Solution of a Many-Fermion System and its Associated Boson Field*," J. Math. Phys. **6**, 304 (1965)

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very same boundary conditions or physical requirements, *e.g*., the filling of the sea of negative-energy particles, that originally had been finessed.[7]

The main difference between the physical and all other, unphysical, solutions was already briefly addressed in footnote 5 of our paper.[6] It lay in the correct choice between two inequivalent representations of creation/annihilation operators.

This is not a mere theoretical "nicety." Many seemingly novel features flow out of a mathematically rigorous, physically acceptable, solution of Luttinger's model that might not have seemed believable if they had been derived from approximate solutions. Among these, one notes correlation functions that satisfy power laws with exponents that depend on the strength of the two-body interactions. Also, the spin and the charge of the original fermions are strictly decoupled and travel at distinct speeds. Similar properties, that once might have been dismissed as unusual offshoots of an unusual model, have now been identified in *all* physically sensible systems of interacting fermions in 1D and thus have the aura of universality that helps distinguish one-dimensional systems (now dubbed "Luttinger Liquids") from the "Fermi Liquids" in higher dimensions.

Subsequently, even as other, ever more complex, models of interacting fermions were analyzed, including E.H. Lieb and F.Y. Wu's remarkable solution of the one-dimensional Hubbard model,[8] Luttinger's remains the gold standard in the field by virtue of its simplicity and general applicability. The present volume celebrating its Jubilee is a fitting tribute to its versatility and should serve as the springboard to its Centennial.

[7] It was fun to find that in addition to the original exact but unphysical solution, Luttinger's model[3] *also* admits *other* exact (equally unphysical) solutions such as the "sausage" in D.C. Mattis and B. Sutherland, "*Strange Solutions*…," J. Math. Phys. **22**, 1692 (1981). Such are the vagaries of infinite sets of coupled linear differential equations.

[8] E.H. Lieb and F.Y. Wu, Phys. Rev. Lett. **20**, 1445 (1968). For additional references and reprints see chapter 4 in the successor volume to ref. 1: D.C. Mattis, *The Many-Body Problem*, *an Encyclopedia of Exactly Solved Models in One Dimension,* World Scientific Publ. Co., Singapore, 1993, 1994, 2009. It is notable that the number of reprints included with that chapter swelled from 3 originally (with just the one[4] on the Luttinger model) to 19 in the successor volume (including seven *specifically* on that topic.)

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